

# Does Kant's conception of Mathematics as “knowledge *a priori*” imply the existence of God?

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# Kant's conception of Mathematics

“A new light must have flashed on the mind of the first man (Thales, or whatever may have been his name) who demonstrated the properties of the isosceles triangle. For he found that it was not sufficient to meditate on the figure, as it lay before his eyes, or the conception of it, as it existed in his mind [...] but that it was necessary to produce these properties, as it were, by a positive *a priori* construction; and that, in order to arrive with certainty at a priori cognition, he must not attribute to the object any other properties than those which necessarily followed from that which he had himself, in accordance with his conception, placed in the object.”

Immanuel Kant, *The Critique of Pure Reason*, Preface to the 2. Edition, 1787, B XI-XII

# Kant's conception of Science

“A much longer period elapsed before physics entered on the highway of science [...]

When Galilei experimented with balls of a definite weight on the inclined plane, when Torricelli caused the air to sustain a weight which he had calculated beforehand to be equal to that of a definite column of water [...] a light broke upon all natural philosophers. **They learned that reason only perceives that which it produces after its own design;”**

Immanuel Kant, *The Critique of Pure Reason*, Preface to the 2. Edition, 1787, B XIII-XIV

**According to Kant  
mathematics and the  
mathematical description of  
nature is something existing *a  
priori* in some mind  
independently of experience.**

**He seems to suggest that this mind is a human  
one (the mind of Thales, Galilei, Torricelli, etc.)**

# Unsolved solvable problems

**Consider the set of natural Numbers,  $N$ :  $\{0,1,2,3,4,\dots\}$ . Different properties, such as *even* number, *prime* number, etc can be defined by a finite number of instructions.**

# Even numbers

A even number is a natural number  
divisible by 2

computer program (capable of deciding  
whether a given number is even or not)

Output: {0,2,4,6,8,10,12,.....}

# Are there infinite even numbers?

Or: Is there a largest even number  $n$ ?

Build the number  $2n$

$2n$  is an even number!

$2n > n$       q.e.d

# Prime numbers

A prime number is a natural number, other than 0 and 1, that is divisible only by 1 and itself.



Computer program



Output: {2,3,5,7,11,13,.....}

# Are there infinite prime numbers?

Or: Is there a largest prime number  $p$ ?

Suppose there is only a finite set of prime numbers:  $\{2, 3, 5, 7, \dots, p\}$

Build the number  $K = 2 \times 3 \times 5 \times \dots \times p + 1$

***K is a prime number***  
***K > p      q.e.d.***

# Perfect numbers

A number is perfect, if it is the sum of all its factors except itself.

$$\text{Example: } 28 = 1+2+4+7+14$$

$$24 \neq 1+2+3+6+8+12 = 32$$

Program

Output: {6,28,496,8128,.....}

# Are there infinite perfect numbers?

**Or: Is there a largest perfect number  $p$ ?**

Until today nobody knows the answer to this question!

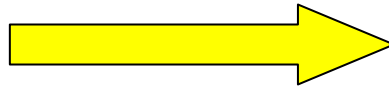
As Hilbert suggests, both, the answer and the method exist.

The answer exists and is yes or no. This answer is knowable, but unknown.

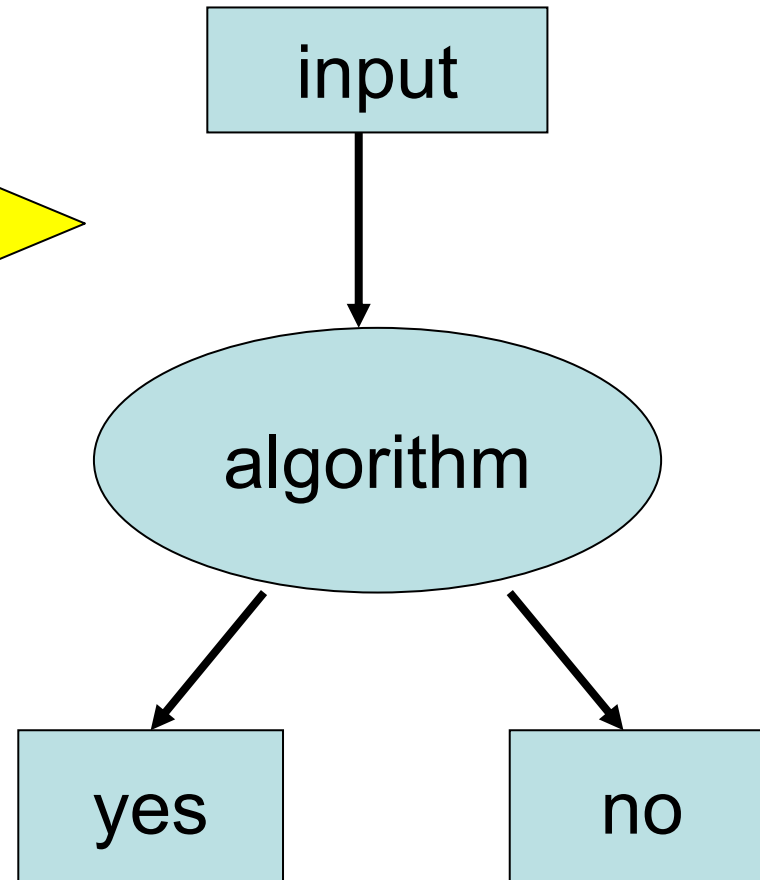
# Decision through mere computing

Is number  $n$

- an even number?
- a prime number?
- a perfect number?



- $S_1: \{0, 2, 4, 6, 8, \dots\}$   
 $S_2: \{2, 3, 5, 7, 11, \dots\}$   
 $S_3: \{6, 28, 496, 8128, \dots\}$   
 $S_i$

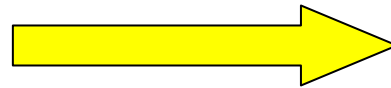


# Decision Problem

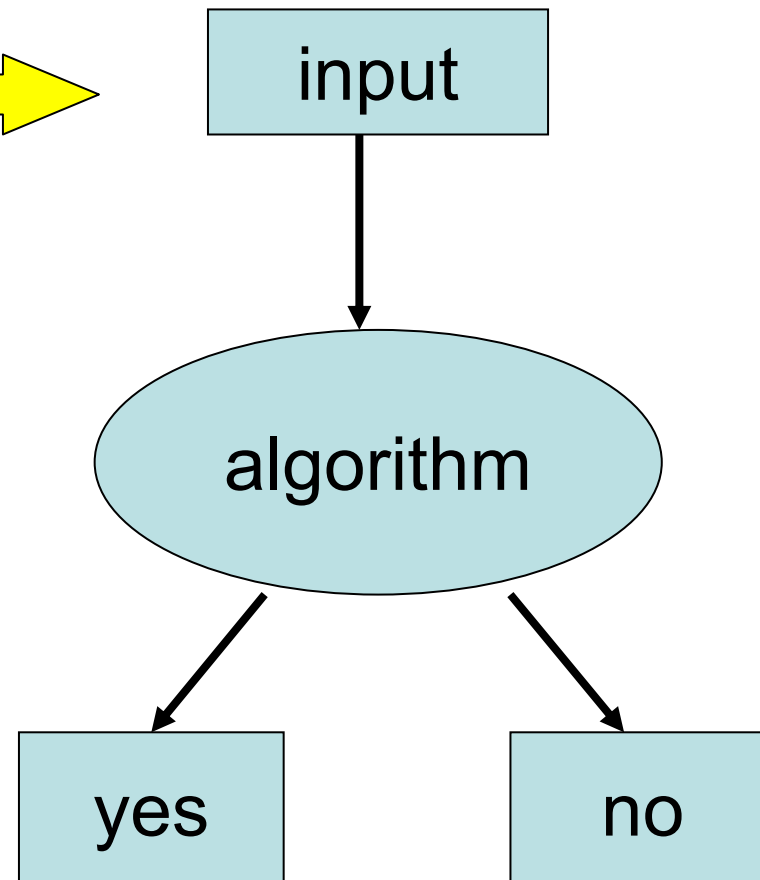
## Decision through mere computing ?

Are there infinite

- even numbers?
- prime numbers?
- perfect numbers?



Program U



Theorem: There is no program  $U$  which allows one to decide, for all  $S_i$ , whether  $S_i$  has a finite or an infinite number of elements.

**In Mathematic there will**  
**always be unsolved**  
**solvable problems**

The proof of the Theorem has  
been given in previous talks

**Conclusion : Mathematical truth will never be completely contained in any human mind.**

- If Mathematics exists *a priori*, like Kant states,

But it is not completely contained in a human mind:

**=> It must be contained in an other mind,  
wich is mightier than the human one,  
i.e. in a omniscient mind.**

- In this sense once can say that the  
existence of God follows from Kant's  
“Critic of pure reason”**