PROOF OF GÖDEL THEOREM

Let's consider the list of all the well formed propositions of a single argument.

We will indicate them with the notation R_k.

The proposition R_k which operates on a specific argument m is denoted by $R_k(m)$.

For example, if the proposition $\forall m[m+1>m]$ is the third proposition of the list, then $R_3(5)$ is the same proposition specified for m=5, i.e. 5+1>5.

We can now write down the list of all the propositions $R_k(m)$:

$R_1(1)$	$R_1(2)$	$R_1(3)$	……	$R_1(m)$	……
$R_2(1)$	$R_2(2)$	$R_2(3)$	……	$R_2(m)$	……
$R_3(1)$	$R_3(2)$	$R_3(3)$	……	$R_3(m)$	……
.	.	.		.	
.	.	.		.	
.	.	.		.	
$R_k(1)$	$R_k(2)$	$R_k(3)$	……	$R_k(m)$	……
.	.	.		.	
.	.	.		.	
.	.	.		.	

Let's consider only the propositions belonging to the diagonal, that is to say the propositions $R_n(n)$.

Since for the diagonal elements $m = k$, these propositions can be characterised by a single natural number.

Let K be the subset of the natural numbers n with the property that $R_n(n)$ is not formally provable.

In other words, n belongs to K if there exists no formal proof of $R_n(n)$.

We can now consider the proposition of the formal system
' n belongs to K ' .This proposition is a well formed proposition of a single argument (as the ones in the list).
So, it must belong to the list, that is to say there exists a number q such that, for all n, $R_q(n)$ is the proposition asserting that n belongs to K.

$R_q(n)$ asserts ' n belongs to K '

Let's consider the proposition of the formal system
$R_q(q)$
Gödel shows that $R_q(q)$ is undecidable but true!

We have seen that

1) **n belongs to K if $R_n(n)$ is not formally provable**
2) **$R_q(n)$ asserts 'n belongs to K'**

Undecidability of $R_q(q)$

If $R_q(q)$ is formally provable, it must be true, and so q must belong to K, that is to say $R_q(q)$ must not be formally provable.
If $R_q(q)$ is formally disprovable, it must be false, and so q must not belong to K, that is to say $R_q(q)$ must be formally provable.

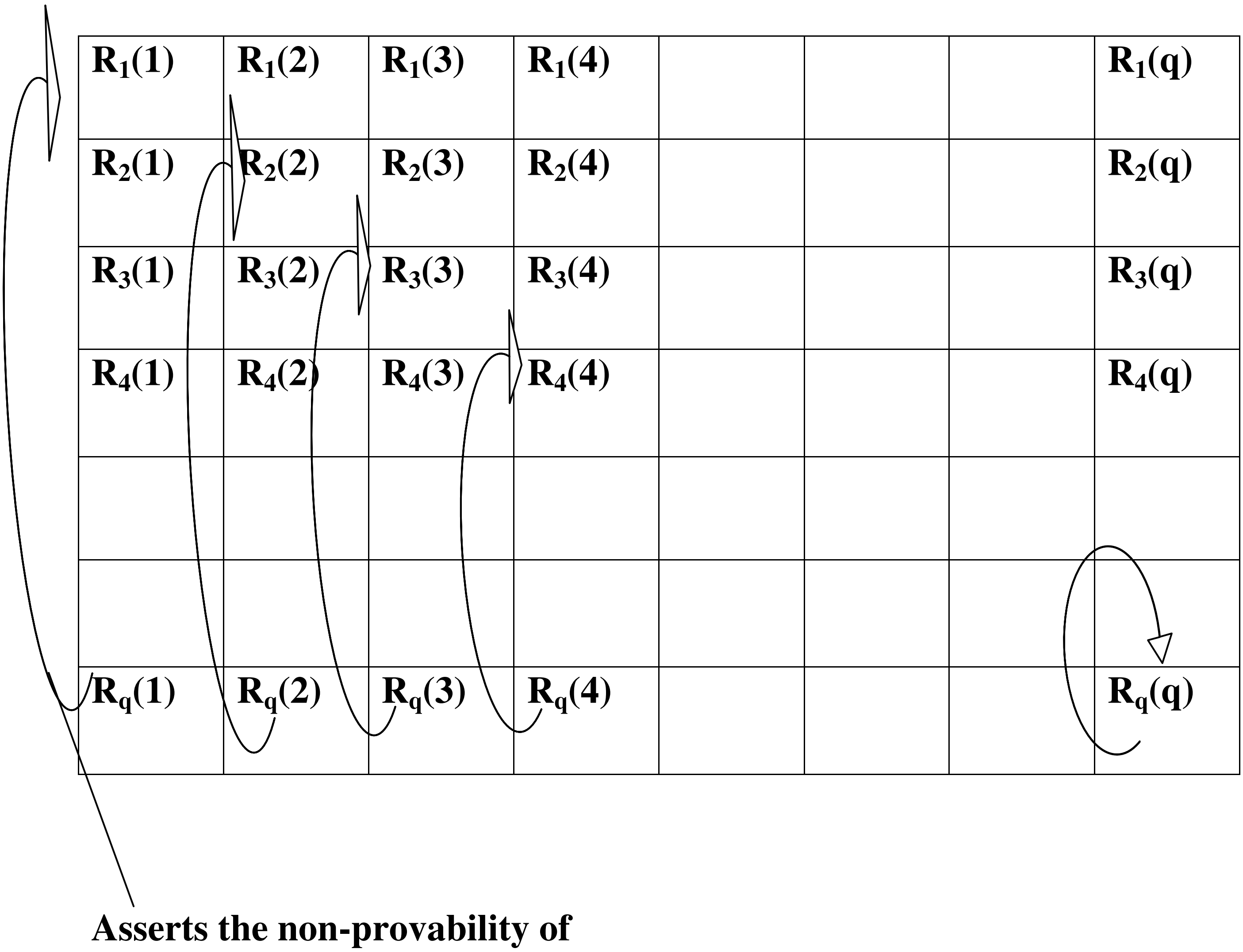
So, $R_q(q)$ is neither formally provable, nor formally disprovable, therefore it is undecidable.

1) **n belongs to K if $R_n(n)$ is not formally provable**
2) **$R_q(n)$ asserts 'n belongs to K'**

Truth of $R_q(q)$

If $R_q(q)$ is false, q must not belong to K, and so $R_q(q)$ must be formally provable and therefore it must be true. This is in contradiction with the premise.
If $R_q(q)$ is true, q must belong to K and $R_q(q)$ must not be formally provable. This was shown before, hence $R_q(q)$ must be true.

Hence $R_q(q)$ is true.



R1(1)
R1(2)
R1(3)
R1(4)
R1(q)
R2(1)
R2(2)
R2(3)
R2(4)
R2(q)
R3(1)
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Rq(1)
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Rq(q)
Asserts the non-provability of