P is not a recursive set: proof

Let's consider the propositions A(n) of our formal system F

A(n) : The nth Turing machine T_n will stop.

Such propositions represent some problems of the arithmetic and they are true for certain values of n and false for all the other values of n.

For example one of these propositions can be:

'the set of the prime numbers is finite'

and we know that this proposition is false.

Turing theorem states that there exists no algorithm that is able to provide a list of all the Turing machines that won't stop, i.e. there exists no algorithm listing all the false A(n), in other words:

Turing theorem states that the set of the false A(n) is not recursively enumerable.

Let’s consider now the true A(n):

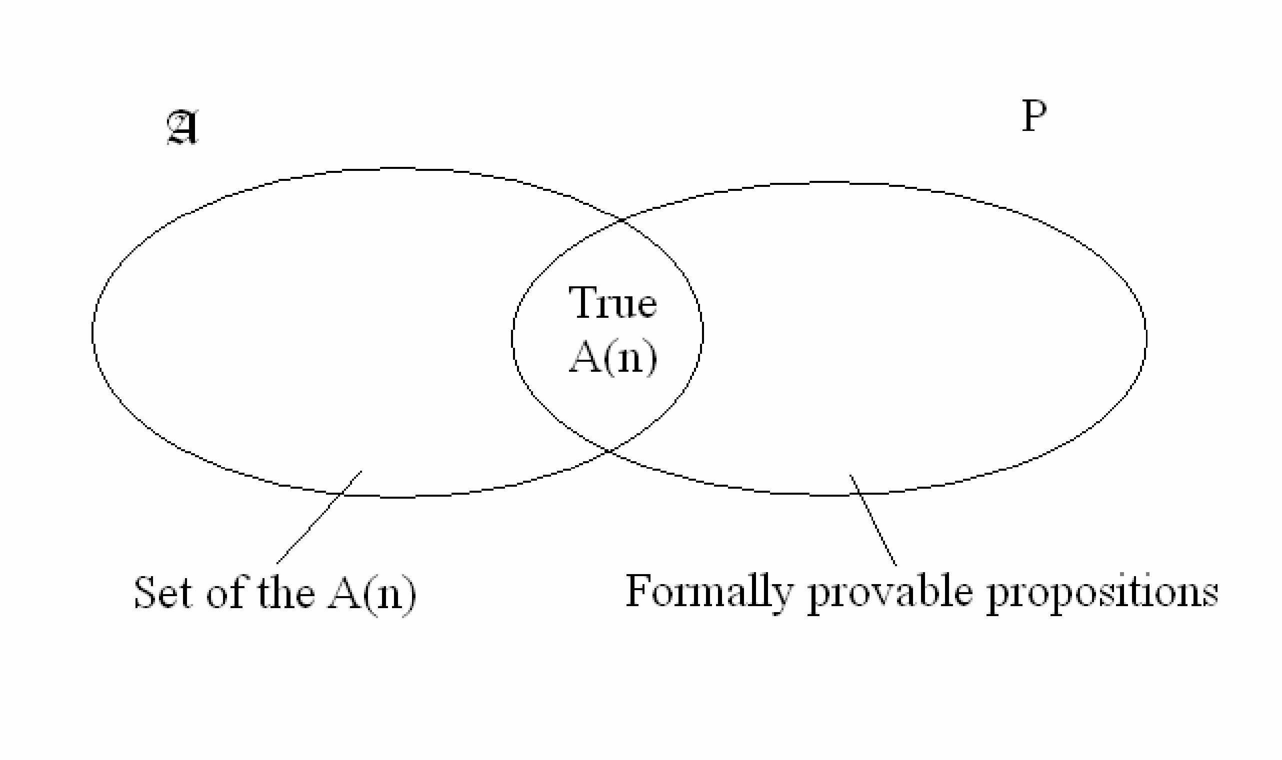
If A(n) is formally provable (that is to say A(n) belongs to P) then it is true, as the formal system is consistent.

If A(n) is true, then Turing machine T_n will stop, and this proposition is formally provable. We only need to follow the operations executed by the T_n and when the T_n stops, A(n) will be proved.

Hence:

A(n) is true if and only if A(n) is formally provable (A(n) belongs to P).

We can represent the situation in this way :



The set of the false A(n) is a subset of P^C and since the set of the false A(n) is not recursively enumerable, even the set P^C is not recursively enumerable.
The set P is recursively enumerable but its complement set P^C is not recursively enumerable, hence

The set P is not recursive.

Summarizing:

TURING THEOREM ⇒ P NOT RECURSIVE ⇒ GÖDEL THEOREM