MIND AND COMPUTERS

Does a computer have a mind?

GÖDEL'S INCOMPLETENESS THEOREM

TRUTH AND DEDUCTION

At school we learned the system of the Euclidean geometry.

There are 5 **axioms** (which define what is meant for point, line …) that are “clearly” true, from which new propositions, called **theorems**, are **deduced**, through necessary **proofs**.

If the rules of logic deduction are followed correctly, these new propositions are true.

The first catalogue of the rules for the correct logic deduction (syllogism) was given by Aristotle (in the Organon). Each subject that is built in a deductive way is impregnable.

THE FORMALIST PROGRAM

In the XIX century, formalists tried to build for every science a corresponding **formal system**, that is to say axioms and rules of deduction, from which **every true proposition** of that science could be necessarily inferred. This property of the system is called **completeness**.

Of course, the axioms at the basis of our system must be **consistent**, i.e. it is not possible to deduce contradictions from them.

FORMAL SYSTEM

A formal system is a **finite collection of symbols** and **precise rules** in order to manipulate these symbols and form combinations (propositions) with them. The correct following of these rules defines the **“well formed”** propositions.

These rules are completely explicit and can be codified into a computer.

The **meaning** of the symbols has no longer importance and the deduction becomes a formal game.

THE DREAM OF FORMALISM

Stating the truth of a proposition of the system would just become a matter of computing. Any other way to estabilish the truth of a mathematical proposition could be brought back to a mere mechanical and formal procedure.

But is that really possible ?

GÖDEL'S INCOMPLETENESS THEOREM, 1930

In every consistent formal system, there always exists a proposition which is neither formally provable nor formally disprovable, yet it's true.

EXAMPLE OF FORMALIZATION

Appendix 1, Empirical evidence for the non material nature of consciousness, J.M. Schins

The stating that can be expressed in natural language as

‘for each number m, m+1 is bigger than m’

can be written in formal language as follows

$$\forall m[m+1>m]$$

using a finite alphabet of symbols as:

$\forall$ m [ ] +1 >

ENUMERATION OF THE PROPOSITIONS

The set of all the propositions of the system is countable, that is to say the propositions can be listed :

R_1

R_2

R_3

……..

R_n

……..

PROPOSITIONS

For example, the proposition

$$\forall m[m + 1 > m]$$

is the R_n for a certain number n, called its Gödel number.

- To each proposition it is possible to associate unambiguously one natural number (its Gödel number) and only one proposition is associated to each natural number.

FORMAL PROOF

A proposition is formally provable if it can be mechanically deduced from the axioms through precise rules.

If a proposition is formally provable, then it's true.

PROPOSITIONAL FUNCTIONS OF A SINGLE ARGUMENT

Let's consider the propositions of a single argument. For example:

$$\forall x\left[(x+1)^2 = x^2 + 2x + 1\right]$$

$$\forall m\left[m+1 > m\right]$$

are propositions of a single argument (x in the first case, m in the second one).

PHILOSOPHICAL IMPLICATIONS

- Mathematical truth and formal deduction are different concepts!
- <<The object of mathematics is to discover “true” theorems.>> (P. Cohen).
- There can be no mathematics without a mind.

THE POWER OF HUMAN MIND

- Gödel’s theorem is often considered as a limit to the mathematical reasoning.

  Instead it is just the opposite:

  the intuitive perception (insight) through which we have stated the truth of the Gödel’s proposition is a reasoning of the human mind not repeatable by a calculator (*Penrose, The Emperor’s new mind*)

SYNTAX AND SEMANTICS

- The way we have ascertained the truth of the Gödel's proposition is through a
  meta-mathematical reasoning reflecting on the meaning of that proposition.
- Syntax will never be able to exhaust semantics. We will never get rid of understanding. Understanding is a human mind's property and not a computer's.

SPIRITUAL ACTS

- The process of our mind throughout we established the truth of Gödel's proposition follows a **reflection principle,**

  We have reflected on the meaning of a proposition that is self-referencing
- Classical philosophical proofs of the spirituality of the human soul are the observations of self-reflecting acts, i.e. I can be aware that I see, that I want, that I think…
- Matter cannot account for these operations.