
DOES MATHEMATICS REQUIRE MIND?

Philosophical consequences of
Gödel's incompleteness theorem

Insight in the truth

- There was always a dream of man to find as much truth as possible and to be sure of it
- Definition 1: The insight of a mind that some statement is certainly true is called PROOF
- There are many kinds of proofs, but we can say that the only real proofs are mathematical proofs, and we will talk about these

Insight in the truth

- Example: Physical statements cannot be proved according to D1 because our mind can't be sure that we are not always dreaming or hallucinating. Proved statements must be proved only by the mind, not by the senses.
- D1 is a philosophical definition, not mathematical

What is mathematics?

- Mathematics is the science of proving, i.e. the science which finds statements which are certainly true
- We may say that mathematics is nothing more than the science of obvious truth. Here obvious means that we are sure of its truthfulness (it has a proof), even if the proof is hard.
- It may seem that there is nothing interesting here, but the fact that something obvious can be such beautiful and complex is fascinating to me

Formal proof

- As we have seen in the previous presentation, there was an attempt to define proof mathematically:
- Definition 2: A statement is **FORMALLY PROVABLE** if it can be mechanically deduced in a formal system.
- This definition doesn't need a mind

Gödel's incompleteness theorem

- Theorem: In every consistent formal system, there always exists a statement which is true but not formally provable
- Definition 3: Let's call that statement a **GÖDEL'S STATEMENT** of that formal system

The power of the mind

- Gödel's theorem is often considered as a limit to mathematical reasoning.
- Instead it is just the opposite: the intuitive perception (insight) through which we have stated the truth of Gödel's statement is a reasoning of the human mind not repeatable by a computer (*Penrose, The Emperor's new mind*)

Consequences

- Mathematical truth and formal provability are different concepts!
- The way we have ascertained the truth of Gödel's statement is through a metaformal reasoning reflecting on the *meaning* of that statement
- We will never get rid of *understanding*
- Understanding is a mind's property and not a computer's

Conclusion

Does mathematics require mind?

Yes!