

On the axiom of choice

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Abstract We describe the famous axiom of choice in mathematics which stands at the very basis of several important branches of modern mathematics. We explain the criticism to the intuitive classical set theory to avoid absurd sets. On the other hand, the tentative to avoid the axiom of choice to define sets just by means of purely mechanical procedures (general recursive functions) is dismissed by Turing Theorem. We describe the Gödel-Cohen undecidability result of the axiom of choice in the Zermelo-Fraenkel axiomatic set theory. We try some philosophical conclusions about the status of the axiom of choice and on the existence of real numbers which are not computable. This can shed some further light on the relations between "randomness and free will".

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